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ABSTRACT

Some of the test questions for the National Council of Architectural Registration Boards deal with the site, including drainage, regrading, and the like. Some questions are most easily scored by examining contours, but others, such as water flow questions, are best scored from a grid in which each element is assigned its average elevation. This document describes a method for creating an elevation grid from a set of contour lines. The general objective is to calculate the perpendicular distances from the center of a grid element to two nearby contours of differing elevation and to assign an elevation to the grid element based on the relative distances from each contour. An appendix displays such a determination. (Contains 12 figures.) (SLD)

RESEARCH REPORT

AN ALGORITHM FOR CONVERTING CONTOURS TO ELEVATION GRIDS

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Educational Testing Service
Princeton, New Jersey
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Abstract

Some of the test questions for the National Council of Architectural Registration Boards (NCARB) deal with the site, including drainage, regrading, and the like. A site is described by its shape and by contours shown on the plan view. Existing structures may be shown on the site. Some questions are most easily scored by examining contours, such as determining if, in regrading the site, the student changed a contour, causing it to pass through an existing structure. However, water flow questions are best scored from a grid of elevations. This document describes a method for creating an elevation grid from a set of contour lines.

AN ALGORITHM FOR CONVERTING CONTOURS TO ELEVATION GRIDS

Introduction

While some NCARB site problems are best solved from the contour map, others are much easier to solve from a grid in which each element is assigned its average elevation. A typical site plan may look like fig. 1, below, in which the boundary of a site surrounds

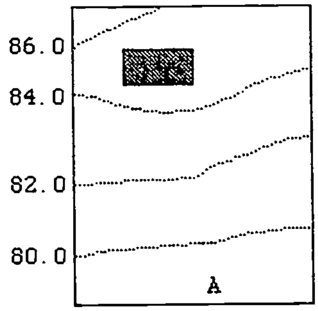

Figure 1. Site With Contours

existing objects and a set of contours defining lines of equal elevation, or height.

Some test questions may refer to water flow on the site. For example, the point A in fig. 1 defines the location of a catch basin, and it is the candidate's task to change the contours (that is, to grade the site) in such a way as to cause water from any point on the site to flow to the catch basin. There is a fairly straightforward way to determine if all locations run downhill, but it requires a matrix of "spot elevations," that is, a two-dimensional array of elevations at regularly spaced intervals over the site, as in fig. 2. A method of converting from contours to an elevation grid is described below.

86.0	86.0	85.9	85.7	85.4	85.0
85.1			84.5	84.2	84.1
83.9	84.0	83.9	83.8	83.2	83.0
82.3	82.1	82.0	82.0	82.0	81.8
81.0	80.8	80.6	80.4	80.3	80.2
79.5	79.4	79.3	79.2	79.2	79.1

A

Figure 2. Site Elevation Grid

Assumptions and Requirements

It is assumed that a set of NBR_c contours exists, numbered k=1 through NBR_c, consisting of connected straight line segments with endpoints $X_{j,k}$, $Y_{j,k}$, j varying from 1 to 1 + the number of segments in the kth contour, and that if the contour is closed, the first endpoint has the same coordinates as the last. It is required to calculate the assumed average elevation of each of an arbitrarily-sized set of square elements $E_{m,n}$ by interpolation from the nearby contours or edges of the site.

Method

Grid Element Calculation: The general objective is to calculate the perpendicular distances from the center of a grid element to two nearby contours of differing elevation, and to assign an elevation to the grid element based on the relative distances from each contour. For example, in fig. 3, the elevation at the center of grid element A is determined by finding the perpendicular distance D_1 from the center of A to contour E_1, the

perpendicular distance D_2 from the center of A to contour E_2, and applying eqn. 1.

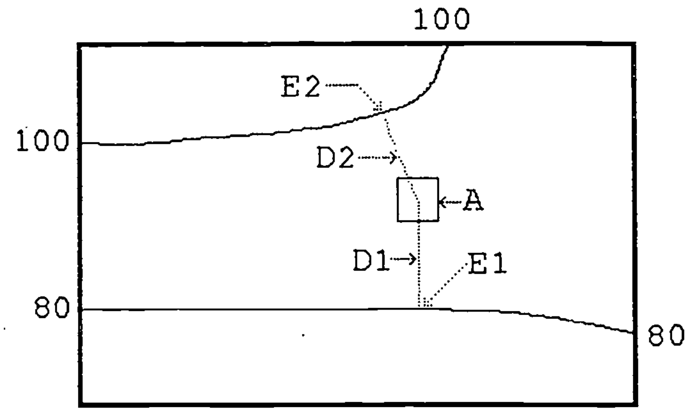

Figure 3: Determining Elevation Of A Grid Element

(1)

$$E_{m,n}=D_1\frac{E_2 - E_1}{D_2 + D_1} + E_1$$

That is, the ratio of the total distance from the two contours to the distance from one contour to point A is used to approximate the elevation at A. Unfortunately, this is only the most straightforward of several possibilities.

In the following examples, let us assume that a search for the closest contour follows only horizontal and vertical lines. Fig. 4 illustrates the difficulties of calculating grid elevation on a hill or in a valley. Case 1 horizontally intersects the same contour on both sides. However, in the vertical direction, contours 140 and 120 are intersected, so eqn. 1 can be used.

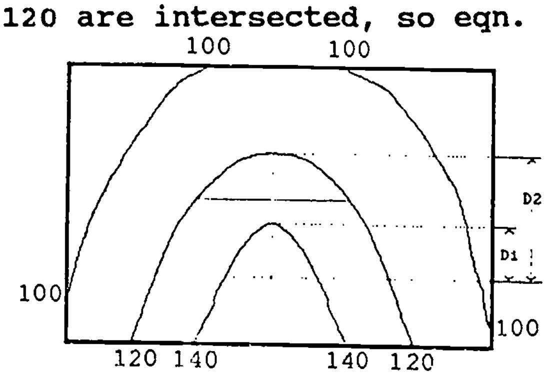

Figure 4. Grid Elevation In A Valley

Case 2 also intersects the same contour horizontally, while vertically, contour 140 is intersected at the top and the edge of the site at the bottom. In this case it is necessary to use contours 140 and 120, even though a contour was crossed to get to 120. While it is possible to use data from contours impinging on an edge, this is probably less accurate.

The physical situation is as shown in fig. 5. Viewed in cross-section, we see that the unknown elevation site is higher than 140.

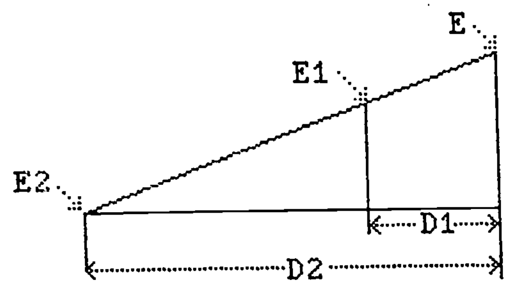

Figure 5. Estimating An Elevation From Two Contours On The Same Side Of The Unknown Elevation

The distance $D_2 - D_1$ is proportional to $E_2 - E_1$, and $E - E_1$ is to D_1 as $E_2 - E_1$ is to $D_2 - D_1$. That is,

(2)

$$\frac{E_2 - E_1}{D_2 - D_1} = \frac{E - E_1}{D_1}$$

or

(3)

$$E = E_1 - D_1 \frac{E_2 - E_1}{D_2 - D_1}$$

Now, we can apply the proportional calculation as in eqn. 3, using the vertical distances from point 2 in fig. 4.

Figure 6 illustrates cases 3 and 4. Either a horizontal or a vertical extension can be used in case 3, but the shorter one

is probably more accurate.

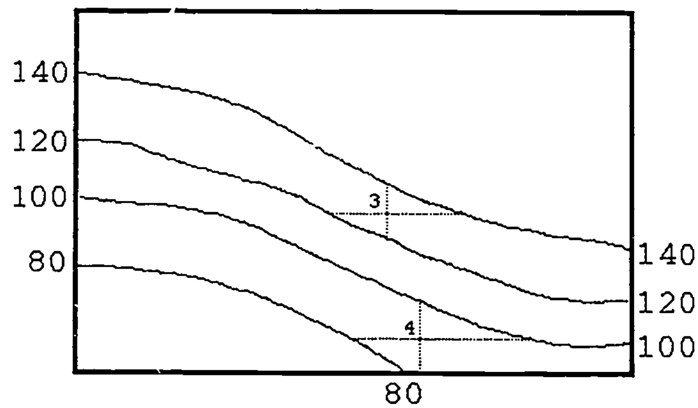

Figure 6. Cases 3 And 4

In case 4, the horizontal extension should be used, because even though it is longer, the vertical extension encounters an edge rather than a contour.

The last three cases to be examined are illustrated in fig. 7.

Case 5 should not be allowed, because the horizontal line connects to no contours and the vertical, even if extended to the top of the site, only contacts contours having the same elevation, namely 80.

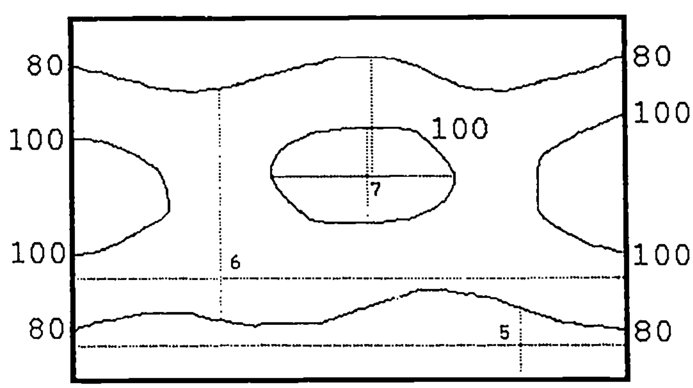

Figure 7. Cases 5, 6, And 7

Case 7 is the same as case 2. Perpendicular lines to two contours must be determined and applied as in eqn. 3.

<u>Determining Perpendicularity at Segment Intersections:</u> A contour is actually represented by a series of connected straight lines, not by a smooth curve. In reality, however, a contour would be smooth, so some opportunities for finding a perpendicular from a grid point to the contour are not apparent. For example, where two non-collinear straight lines meet, there is really a curve near the intersection. (See fig. 8.)

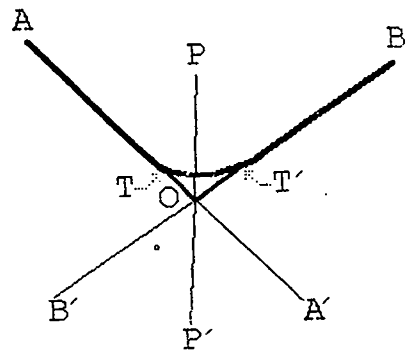

Figure 8. Actual Contour And Approximation

Contour segments AO and OB are shown in fig. 8 overlaying the true contour, that is, the contour represented by a curve. It is assumed that the actual curve includes a circular arc tangent to AA′ at T and to BB′ at T′. A line from the grid point P or P′ to the point O, has a normal through O that is tangent to the arc only if the normal line ON (see fig. 9) does not intersect the line AB between A and B. This can be determined by comparing the area of triangle AOB with triangles AON and BON. If AON + BON $\neq$ AOB, then the intersection is outside AOB. In fact, it is easier to compute the areas of trapezoids

(4)

$$\begin{aligned} AOB &= ABX_3X_1 - AOX_2X_1 - BOX_2X_3 \\ AON &= ANX_4X_1 - AOX_2X_1 - NOX_2X_4 \\ BON &= BOX_2X_3 + BNX_4X_3 - NOX_2X_4 \end{aligned}$$

Note that X_1 is the X-value of the point A, X_2 of O, X_3 of B, and X_4 of N.

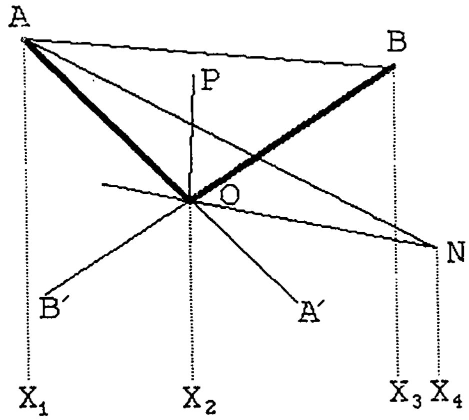

Figure 9. Determination Of Intersection Of Lines ON And AB

The area of a trapezoid can be computed from the X,Y values of the four points defining the trapezoid. The Y-location of points X_1 through X_4 can be any arbitrary value, so for simplicity we can let it be zero. In terms of the X,Y locations of the corners of the trapezoid ABX_2X_1, we need to calculate the area of the rectangle BCX_1X_2 plus the triangle ABC. (See fig. 10.)

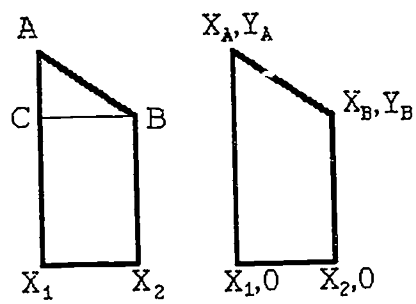

Figure 10. Calculating Area Of A Trapezoid

So the area of the rectangle is $(X_2 - X_1)Y_B$ and the area of the triangle is $(X_2 - X_1)(Y_A - Y_B)/2$, so the area of the trapezoid is

(5)

$$
\begin{aligned}
& Y_B(X_2 - X_1) + \frac{(X_2 - X_1)(Y_A - Y_B)}{2} \\
& = (X_2 - X_1)(Y_B + \frac{(Y_A - Y_B)}{2}) \\
& = (X_2 - X_1)\frac{(Y_A + Y_B)}{2}
\end{aligned}
$$

Injudicious choice of X_1 and X_2 can cause the result of eqn. 5 to be negative. Since this is an area, however, it can only be a non-negative number, so it is legitimate to take the absolute value as the correct result.

The next question is how to determine if the intersection of lines AB and ON (the point C in fig. 11) is between A and B. We can determine the equation of the line ON and then find the point C that is the intersection of ON and AB. The equation of OP is

(6)

$$\frac{X_C - X_O}{X_P - X_O} = \frac{Y_C - Y_O}{Y_P - Y_O}$$

the slope of OP is

(7)

$$M = \frac{Y_P - Y_O}{X_P - X_O}$$

so, since the slope of a perpendicular is -1/M, the equation of OC is

(8)

$$Y_C - Y_O = \frac{-1}{M}(X_C - X_O)$$

and the equation of AB is

(9)

$$\frac{X_C - X_B}{X_A - X_B} = \frac{Y_C - Y_B}{Y_A - Y_B}$$

Solving eqns. 8 and 9 simultaneously for Y_C results in

(10)
$$Y_C = \frac{MY_O + X_O + \dfrac{Y_B(X_A - X_B)}{Y_A - Y_B} - X_B}{M + \dfrac{X_A - X_B}{Y_A - Y_B}}$$

And X_C can then be found from

(11)
$$X_C = \frac{Y_C - Y_B}{Y_A - Y_B}(X_A - X_B) + X_B$$

As usual, there are cases needing special handling. Equations

8 through 11 cannot be applied if $X_P = X_O$, $Y_P = Y_O$, $X_A = X_B$, $Y_A = Y_B$,

or if

(12)
$$\frac{-1}{M} = \frac{Y_A - Y_B}{X_A - X_B}$$

A complete algorithm for determining if a tangent exists is attached as Appendix A.

If the area of triangle AOB equals AOC + BOC, then there is no tangent near O. Doing the same calculation for P′ in fig. 11, we construct the line ON′ and find C′, the point of intersection of ON′ and AB. The area AOB is less than AOC′ + BOC′, and a line from N′ to O will be tangent to the contour.

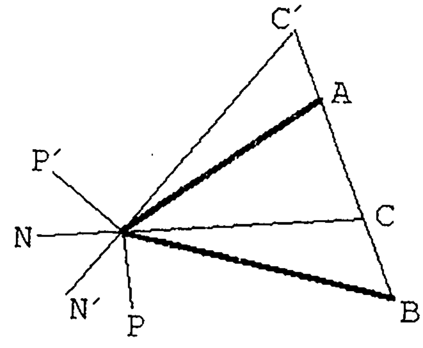

Figure 11. Solving For C Inside Triangle AOB

<u>Determining Perpendicularity to Contour Segments:</u> Next we consider perpendiculars to contour line segments. The endpoints of a contour line segment are X_A, Y_A and X_B, Y_B. This line has a slope of M. If there is a perpendicular to the line, the perpendicular from X_P, Y_P will have slope -1/M and will intersect the contour segment at X_C, Y_C such that X_C and Y_C are between the endpoints of the contour segment. We find X_C and Y_C from:

(13)

$$\frac{X_C - X_A}{X_B - X_A} = \frac{Y_C - Y_A}{Y_B - Y_A}$$

$$M = \frac{Y_B - Y_A}{X_B - X_A}$$

$$Y_C - Y_P = \frac{-1}{M}(X_C - X_P)$$

If $X_A = X_B$, the line is vertical. In this case, if $X_P = X_B$, there is no perpendicular. Otherwise, we set $X_C = X_B$ and if X_C and Y_C are between the endpoints of the segment, a perpendicular exists.

If $Y_A = Y_B$, the line is horizontal. We set X_C to X_P and Y_C to Y_B, and if X_C and Y_C are between the endpoints of the segment, a perpendicular exists.

If the segment is neither horizontal nor vertical, we compute

(14)

$$WK_2 = \frac{X_B - X_A}{Y_B - Y_A}$$

and now we can compute Y_C and X_C:

(15)

$$Y_C = \frac{Y_A WK_2^2 + (X_P - X_A)Wk_2 + Y_P}{WK_2^2 + 1}$$

$$X_C = (Y_C - Y_A)WK_2 + X_A$$

contour segment by

(16) $$(X_A \le X_C \cap X_B \ge X_C) \cup (X_A \ge X_C \cap X_B \le X_C)$$

If the expression in eqn. 16 is false, there is no perpendicular to the segment. If it is true, we then test Y_C:

(17) $$(Y_A \le Y_C \cap Y_B \ge Y_C) \cup (Y_A \ge Y_C \cap Y_B \ge Y_C)$$

If the expression in eqn. 16 is also true, the line from X_P, Y_P to X_C, Y_C is perpendicular to the contour segment.

<u>Determining if a Perpendicular Crosses a Contour:</u> We also need a method to determine if a perpendicular crosses a contour. We have the endpoints of the perpendicular, X_P,Y_P and X_C,Y_C, and we have the endpoints of a contour segment, X_A,Y_A and X_B,Y_B, so we have to solve eqns. 18 simultaneously for the intersection X_I,Y_I:

(18) $$\frac{X_I - X_P}{X_C - X_P} = \frac{Y_I - Y_P}{Y_C - Y_P}$$
$$\frac{X_I - X_A}{X_B - X_A} = \frac{Y_I - Y_A}{Y_B - Y_A}$$

keeping in mind that it is necessary to handle the usual special cases. If $Y_A = Y_B$, then the contour segment is horizontal and if $Y_C = Y_P$, the perpendicular is parallel to the segment and cannot intersect it. If $Y_A = Y_B$ and $Y_C <> Y_P$, then

(19) $$Y_I = Y_P$$
$$X_I = \frac{X_C - X_P}{Y_C - Y_P}(Y_I - Y_P) + X_P$$

If $Y_C = Y_P$, the perpendicular vector is horizontal and

(20)
$$Y_i = Y_P$$
$$X_I = \frac{X_B - X_A}{Y_B - Y_A}(Y_I - Y_A) + X_A$$

Otherwise, we have the case where neither line is horizontal, but we still have to be aware that they may be parallel:

(21)
$$WK_1 = \frac{X_B - X_A}{Y_B - Y_A}$$
$$WK_2 = \frac{X_C - X_P}{Y_C - Y_P}$$

If $WK_1 = WK_2$, the lines are parallel. If not, we can compute Y_I and X_I:

(22)
$$Y_I = \frac{X_P - Y_P WK_2 + Y_A WK_1 - X_A}{WK_1 - WK_2}$$
$$X_I = WK_2(Y_I - Y_P) + X_P$$

Now we must test that X_I and Y_I are between the endpoints of <u>both</u> lines, as in eqns. 16 and 17. If so, the perpendicular vector crosses a contour.

<u>Computational Procedure:</u> To begin the computation process, we must first establish that AO and OB are not collinear. The easiest way to do this is to use the trapezoid method to calculate the area of AOB. If it is zero, the lines are collinear and there is no perpendicular at O.

Now we are ready to build a table of all perpendiculars for element $E_{m,n}$; not only perpendiculars to the lines but also to the intersections, as described above. We will examine all contours

for instances of a perpendicular, save all instances in a table, and then process the table to find the best pair of perpendiculars on which to calculate the elevation $E_{m,n}$. Since perpendiculars that do not cross a contour are preferred, we will flag those that do. The table will contain the lengths of all perpendiculars from an $E_{m,n}$ to all coordinates, the associated elevations, and a flag '-1' if this perpendicular did not cross a contour, or '+1' if it did, and the contour number (that is, the number between 1 and NBR_C that is the contour currently being processed.) When the table is complete, we will sort it on increasing perpendicular length within increasing flag, because we want to use the shortest two perpendicular lengths to contours if we can, giving preference to entries that did not cross a contour.

Now, while there are at least two entries left in the table, we look at the first two entries. If they are to two different contours, we use them to compute the elevation of $E_{m,n}$ and we can now repeat the process for the next E-elevation. If they are the same contour, we delete the second table entry by replacing it with entry 3, entry 3 with entry 4, etc., and repeat from the beginning of this paragraph. If the table reduces to less than two entries an E-elevation cannot be computed.

It is necessary, after determining that there are two usable entries in the table, to decide if the elevation should be computed by eqn. 1 or eqn. 2. Equation 2 must be used if the perpendicular to the second table entry crosses over the contour of the first. Using the contour number from the table, we can

determine if the perpendicular to the second contour crossed over the first contour by the procedure described above.

Suggestions for Further Study

There are cases that depend on the layout of the contours. For example, in fig. 12, the grid point at A calculates to a value of 44, even though it is clearly greater than 100. (E_1 = 120, E_2 = 140, D_1 = 38, D_2 = 48. See eqn. 3.) This is due to the absence of perpendiculars from A to contour 100. If the contour were to be continued outside the top of the site, the calculation for A would be correct, because a perpendicular would be present.

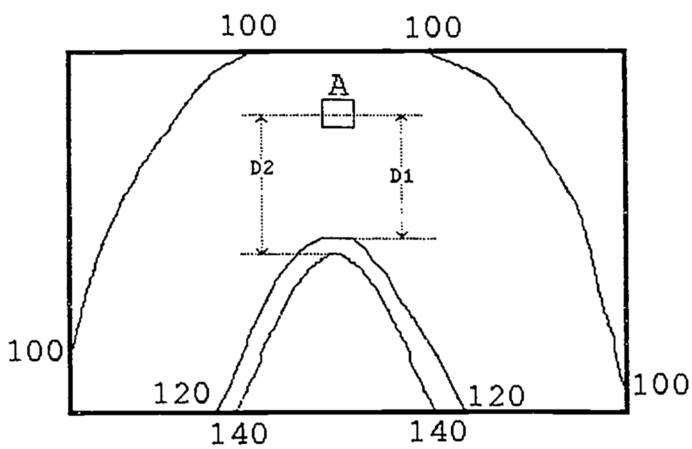

Figure 12. Incorrect grid computation

If continuing broken contours is not permitted, then it would be a good idea to require that all contours be equidistant in terms of their elevations, as in figure 12, where the change in elevation from one contour to the next is uniformly 20 units. Then the program could determine that values of 80 and below, or 160 and above, are not valid.

It is also important to require reasonably close polyline approximation to the original contour, and to have more than two contour lines on a site. Otherwise, it is possible that fewer

than two perpendiculars exist and hence the spot elevation calculation cannot be made.

APPENDIX A

DETERMINATION OF A LINE FROM X_p, Y_p PERPENDICULAR TO A TANGENT THROUGH X_0, Y_0 ON A CIRCULAR ARC